

Spherically symmetric models for charged radiating stars and voids: Theoretical approach

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Abstract

We study the matching of a general spherically symmetric spacetime with a Vaidya-Reissner-Nordström solution. To that end, we study the properties of spherically symmetric electromagnetic fields and develop the proper *gravitational and electromagnetic* junction conditions. We prove that generic spacetimes can be matched to a Vaidya-Reissner-Nordström solution or one of its specializations, and that these matchings have clear physical interpretations. Furthermore, the non-spacelike nature of the matching hypersurface is proved under very general hypotheses. We obtain the fundamental result that any spherically symmetric body, be it in evolution or not, has an upper limit for the total net electric charge that carries.

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1 Introduction

Frequently, within the framework of General Relativity, in order to describe new or interesting physical situations, or to investigate certain theoretical aspects, the construction of a model is required. This is commonly done by matching two given and known space-times. Thus, for instance, models for stars (to study the genesis of black holes, the possibility of escaping a black hole region, the violation of the cosmic censorship conjecture, etc.) and models for local inhomogeneities in a cosmological context (to study the origin of galaxies in the primitive Universe, the possibility of primordial black holes, the evolution of the stars in the Cosmos, etc.) can be studied by means of the junction of two different space-times.

With regard to the first type –construction of stellar models– the common feature in most of them is the matching of two space-times with special characteristics: one of them, which describes the so called *interior* of the star; and another, without matter but possibly with electromagnetic fields and/or null radiation, which represents its *exterior*. Both regions are separated by a timelike hypersurface, representing the surface of the star. A very typical simplifying assumption is that the complete space-time possesses spherical symmetry, so that in order to describe the exterior of an object characterized only by its mass the Schwarzschild vacuum solution is used; if, in addition, the space-time possesses a net electrical charge, then the Reissner-Nordström (RN) solution is used; and if radial null radiation is included then, instead of the previously cited metrics, the solutions of Vaidya [52] and Vaidya-Reissner-Nordström (V-RN) [30], are used, respectively.

Thus, we can classify the works for stellar models according to their *exterior*: The archetype with a Schwarzschild exterior is the pioneering work by Oppenheimer-Snyder [43], in which the collapse of a dust cloud was studied. This work was later generalized from two different points of view: on the one hand, by considering a more general interior matter distribution. This was done for instance by Misner and Sharp [39], where a perfect fluid interior was considered, and by Bel and Hamoui [2] for the case of a fluid with anisotropic pressures. On the other hand, by allowing for the existence of an electromagnetic field. For example, when a charged dust interior with a Reissner-Nordström exterior was considered. Outstanding examples of this treatment were the works by Novikov, de la Cruz & Israel, Markov & Frolov, Hamoui, Vickers, Misra & Srivastava, Raychaudhuri and de Felice & Maeda [41].

Recently, Amos Ori [44] has found the explicit general solution for the Einstein-Maxwell's equations in this case.

The next natural step for stellar models with a RN exterior was to consider an even more general interior. This was initially achieved by Bekenstein [1], who considered a charged perfect fluid interior, by using a generalization of the formalism (for neutral perfect fluids) due to Misner and Sharp. Some other interiors were later considered. This is the case of the article by Florides [19], in which an exact solution of the Einstein-Maxwell equations for a static, spherically symmetric charge distribution is found. This represents an extension, to the charged case, of a previous solution by Synge [49].

With regard to the radiating stars, that is, models with a Vaidya exterior, we have to point out the works by Herrera *et al.* (see [24] for a summary of their results) and Santos *et al.* (see [6] and references therein), where the interior energy-momentum tensor is interpreted as a fluid with heat flux. However, these works study only the shear-free fluid case. Less restrictive is the article by Fayos *et al.* [16] where the preceding works on Schwarzschild and Vaidya [52] interiors were generalized.

The complete case with a radiating and charged exterior (that is, a V-RN exterior) has been treated by Oliveira and Santos [42], but only in the case of a charged fluid interior with heat flux but without shear. This article represents a generalization of the work by Misner and Sharp, and also, of the one by Bekenstein, in which dynamical equations for the spherical collapse are given. Likewise, Herrera and Núñez [24] considered a V-RN exterior, but only for a charged shear-free perfect fluid interior, and with a flux of non-polarized radial radiation. In their article they extend the *HJR method* –described in this same reference– in order to obtain the evolution of radiating charged fluid spheres [38]. They use a heuristic *ansatz* which allows them to integrate the Einstein-Maxwell equations taking into account the matching conditions.

Regarding the construction of local inhomogeneities in a cosmological context, the first fundamental work is the historically relevant paper by Einstein and Strauss [14], in which “the influence of the expansion of space on the gravitation fields surrounding the individual stars” was studied. They considered a vacuum Schwarzschild *interior* surrounded by a Universe, modeled by a Robertson-Walker *exterior*. The generalization, replacing the Schwarzschild solution with a Vaidya interior, in order to describe the so called *primordial black holes*, can be found in [21, 47, 31, 15].

Lake and Hellaby [32] used these models to study the formation of naked singularities in the collapse of radiating stars, as candidates for counterexamples of the cosmic censorship conjecture.

It should be emphasized here that the actual matching procedure for the Einstein-Straus and Oppenheimer-Snyder models are *mathematically identical* and indistinguishable, because the two glued spacetimes are exactly the same: vacuum Schwarzschild with dust Robertson-Walker. They are in fact an outstanding example of *complementary matchings*, see [18]. This is in fact a general feature of the matching procedure and thus we can always attack the two problems (description of stars, and modelization of voids) in a common and unique framework. This is one of our aims in this paper, and to that end we will follow the philosophy put forward in [18] which will allow us to treat the two possibilities jointly in a natural way.

As a second goal of our paper, we wish to treat the fully general case in which the matter content in the interior for stars (or the exterior for voids) is left unrestricted, and the corresponding exterior (resp. interior) can contain both radiation and electromagnetic fields. We will attack this problem by studying the matching of two spherically symmetric space-times \mathcal{V} and $\bar{\mathcal{V}}$ through a time-like matching hypersurface Σ in such a way that one of the space-times will be completely general (\mathcal{V}), and the other ($\bar{\mathcal{V}}$) may contain an electromagnetic field and/or radial null radiation. This means that $\bar{\mathcal{V}}$ will be described by a V-RN solution or one of its special cases. We remark that our results will be general and independent of the different combinations that might exist in $\bar{\mathcal{V}}$: vacuum, only electromagnetic field, only radial null radiation, or radiation and electromagnetic field.

The possible physical justification for including electromagnetic charge in the models arises from several different considerations: i) first of all, in the formation of a stellar object the repulsive Coulomb force acting in every charged particle of the same sign of the net charge of the star cannot be arbitrarily big, as is obvious. A simple calculation, taking into account the opposing forces acting inside a star with n_b baryons, indicates that the net charge Q must obey the inequality $Q < 10^{-36} n_b$ [20]. This means that only a small value of the net charge per nucleus is permitted, but not necessarily zero; ii) the existence of an electromagnetic field has drastic consequences on the global space-time structure –compare, for instance, the Schwarzschild solution with the Reissner- Nordström solution–. Thus, the evolution of a star with a net electric charge Q could be totally different to the evolution of a neutral star, no

matter how small Q may be. New possibilities –like that of a star avoiding the collapse towards a singularity– have been investigated since the appearance of the previously cited article by de la Cruz and Israel, in which it was shown for the first time that a charged object, after collapsing beyond the event horizon, can expand and re-emerge in another asymptotically flat region. Of course, all this depends on the stability of the Cauchy horizons, see e.g [50] and references therein, a problem which is still subject to controversy; finally, even if there is no electromagnetic field in $\bar{\mathcal{V}}$, we could allow for a radial charge distribution in \mathcal{V} with total net charge vanishing. This is necessary to describe charge redistributions that may take place in the interior of a star. A phenomenon like this happens, for instance, in the *hybrid stars*, in which there is global, but not local, charge neutrality, see [20].

The plan of the paper is as follows. First, we will study, in section 2, the properties of general spherically symmetric space-times. In particular we write down the necessary and sufficient conditions which guarantee the absence of curvature singularities. Since we want our models to have a physical meaning, we impose the fulfillment of the dominant energy conditions and, as a first consequence of this, we obtain the conditions such the *mass function* will be non-negative. In section 3 we analyze the electromagnetic fields, both null and non-null, that are compatible with the spherical symmetry, and the restrictions on the distribution of charge that arise at this level. Then we devote section 4 to the study of the spherically symmetric V-RN space-time $\bar{\mathcal{V}}$. In section 5, the gravitational *and electromagnetic* matching conditions are derived. We distinguish the cases with or without a surface charge current density in the matching hypersurface. At the end of this section we study the matching results, their interpretations and some consequences. Next, in section 6, we analyze the restrictions on the matter contents of the space-time \mathcal{V} derived from the matching conditions. In order to guarantee the physical meaning of \mathcal{V} –no matter which interpretation of the matter content is used–, in section 7 we will demand the fulfillment of the dominant energy conditions to the energy-momentum tensor, and we will examine the physical consequences that such constraints imply. We end up with some conclusions, in which the main results are summarized: they are the existence of a maximum for the total charge of the bodies, the matchability of generic spacetimes to V-RN or its specializations –these matchings having a clear physical interpretation–, and the causality of the matching hypersurface in general.

2 Basics on spherically symmetric space-times.

Let us consider a four-dimensional spherically symmetric space-time \mathcal{V} , so that its line-element can be expressed in radiative coordinates [4] $\{x^\mu\} = \{u, R, \theta, \varphi\}$ ($\mu = 0, 1, 2, 3$) as

$$ds^2 = -e^{4\beta}\chi du^2 + 2\varepsilon e^{2\beta} du dR + R^2 d\Omega^2, \quad (1)$$

where $\chi \equiv 1 - 2m/R$, $\varepsilon^2 = 1$, β and m depend on $\{u, R\}$, and $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$. It is easily checked that $m(u, R)$ is the well-known mass function defined by Cahill & McVittie [8], which represents the total energy inside the two-spheres with constant values of u and R , see e.g. [23].

The global structure and general properties of the space-time depend on the behavior of the functions $m(u, R)$ and $\beta(u, R)$. In this sense, for example, it can be seen that no spherical closed trapped surfaces (see [22] for definitions) are present in the whole spacetime if and only if χ is non-negative everywhere (that is to say, $2m(u, R) \leq R$). In fact, the *two-spheres* defined by $u = u_c = \text{const.}$ and $R = R_c = \text{const.}$ are closed trapped surfaces if and only if $\chi(u_c, R_c) < 0$, and thus the hypersurface $\chi = 0$, which is the boundary between regions with and without closed trapped spheres, is the *apparent horizon*.

Similarly, all curvature invariants will be finite at $R = 0$ [51] if and only if (comma denotes partial derivative)

$$\beta_{,R}(0) = 0 ; \quad m(0) = m_{,R}(0) = m_{,RR}(0) = 0$$

so that we have

Proposition 2.1 *The necessary and sufficient conditions preventing the existence of a curvature singularity at $R = 0$ are*

$$\begin{aligned} \lim_{R \rightarrow 0} \frac{\beta(u, R) - \beta_0(u)}{R^2} &= \beta_2(u), \\ \lim_{R \rightarrow 0} \frac{m(u, R)}{R^3} &= m_3(u), \end{aligned} \quad (2)$$

where $\beta_0(u) \equiv \lim_{R \rightarrow 0} \beta(u, R)$, $\beta_2(u)$ and $m_3(u)$ are finite functions of u .

We shall later use these conditions.

In order to build up a model, some knowledge or constraints on the energy-momentum content, evaluated over the region of the space-time under study, are

compulsory. The standard *energy conditions*, see e.g. [22], are among these physical constraints, so that in this paper we will always demand that the space-times under consideration fulfill the *dominant* energy condition (DEC), which requires that the energy-flux 4-vector is always non-spacelike [22]. For spherically symmetric space-times with the metric expressed in radiative coordinates, the DEC reduces to the following inequalities:

$$\beta_{,R} \geq 0, \quad (3)$$

$$m_{,R} - R\chi\beta_{,R} \geq 0, \quad (4)$$

$$\chi^2\beta_{,R} + \varepsilon \frac{2e^{-2\beta}}{R} m_{,u} \geq 0, \quad (5)$$

$$\lambda \geq |P_2|, \quad (6)$$

where

$$\begin{aligned} \lambda &= -\frac{2\chi}{R}\beta_{,R} + \frac{2}{R^2}m_{,R} + \frac{2}{R} \left(\chi^2\beta_{,R}^2 + \varepsilon \frac{2e^{-2\beta}}{R} m_{,u} \beta_{,R} \right)^{1/2}, \\ P_2 &= \left(\frac{3}{R} - \frac{\chi}{R} + 4\chi\beta_{,R} - \frac{6}{R}m_{,R} \right)\beta_{,R} + 2\chi\beta_{,RR} - \frac{1}{R}m_{,RR} + \varepsilon 2e^{-2\beta}\beta_{,uR}. \end{aligned}$$

The inequality (4) can be rewritten as

$$(me^{2\beta})_{,R} \geq \frac{R}{2}(e^{2\beta})_{,R} \geq 0$$

where the last inequality follows from (3). Take an arbitrary null hypersurface $u = u_c = \text{const.}$ By integrating the above inequality in such a hypersurface between R_1 and R_2 ($> R_1$) we obtain

$$m(u_c, R_2)e^{2\beta(u_c, R_2)} - m(u_c, R_1)e^{2\beta(u_c, R_1)} \geq \int_{R_1}^{R_2} \frac{R}{2}(e^{2\beta(u_c, R)})_{,R} dR \geq 0$$

so that

$$m(u_c, R_2)e^{2\beta(u_c, R_2)} \geq m(u_c, R_1)e^{2\beta(u_c, R_1)} \quad (7)$$

From here we deduce that if $m(u_c, R_1) \geq 0$, then $m(u_c, R) \geq 0$ for all $R \geq R_1$. In particular, it is sufficient that $\lim_{R \rightarrow 0} m(u_c, R)e^{2\beta(u_c, R)} \geq 0$ for the mass function to be non-negative on the whole hypersurface $u = u_c$. Taking into account that $u = u_c$ is arbitrary, we have proven the following

Proposition 2.2 *If a spherically symmetric space-time is such that the mass function $m(u, R)$ is non-negative at $R = 0$ and the dominant energy conditions are fulfilled, then $m(u, R) \geq 0$ for all $R > 0$.*

Actually, not all DEC's are needed here, but just conditions (3-4). An interesting case for this proposition, taking into account (2), is the following (compare with the more restrictive *prop. 6* in [23])

Corollary 2.1 *If a spherically symmetric space-time has no curvature singularity at $R = 0$ and the DEC are fulfilled, then the mass function $m(u, R)$ is non-negative everywhere.*

3 Electromagnetic fields with spherical symmetry

Starting with a spherically symmetric space-time in its form (1) we can compute, through Einstein's equations, its energy-momentum tensor \mathbf{T} . Choosing an orthonormal cobasis $\{\mathbf{V}^{(\alpha)}\} = \{\mathbf{u}, \mathbf{n}, \boldsymbol{\omega}_\theta, \boldsymbol{\omega}_\varphi\}$ such that $\boldsymbol{\omega}_\theta \equiv R d\theta$, $\boldsymbol{\omega}_\varphi \equiv R \sin\theta d\varphi$, \mathbf{n} is a space-like 1-form orthogonal to $\boldsymbol{\omega}_\theta$ and $\boldsymbol{\omega}_\varphi$, but otherwise *arbitrary*, and \mathbf{u} is a time-like 1-form which completes the orthonormal tetrad, one gets:

$$\begin{aligned} \mathbf{T} = & A \mathbf{u} \otimes \mathbf{u} + B \mathbf{n} \otimes \mathbf{n} + C (\mathbf{u} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u}) + \\ & + D (\boldsymbol{\omega}_\theta \otimes \boldsymbol{\omega}_\theta + \boldsymbol{\omega}_\varphi \otimes \boldsymbol{\omega}_\varphi) \end{aligned} \quad (8)$$

where A, B, C and D are functions of u and R . This energy-momentum tensor will be, in general, a mixture of fluids, gas, radiation and perhaps an electromagnetic field. For the sake of generality, we shall not identify all the particular fields and components of the matter contents. Nevertheless, avoiding this general rule, we shall in fact identify the electromagnetic part of \mathbf{T} . This will reveal very useful because, among other things, it provides junction conditions for the electromagnetic field itself, independent of the gravitational ones, a fact which plays a central role in the physics of the problem. Thus, throughout this paper we will speak of the spherically symmetric electromagnetic field \mathbf{F} , which is of course a 2-form solution of the Maxwell equations and must be considered as a partial source of the gravitational field so that

$$\mathbf{T} = \mathbf{P} + \mathbf{E} \quad (9)$$

where \mathbf{E} is the part associated to the electromagnetic field

$$\mathbf{E} = \frac{1}{4\pi} (F_{\alpha\gamma} F_{\beta}{}^{\gamma} - \frac{1}{4} F_{\gamma\delta} F^{\gamma\delta} \eta_{\alpha\beta}) \mathbf{V}^\alpha \otimes \mathbf{V}^\beta \quad (10)$$

and \mathbf{P} is the *non-electromagnetic* part. Obviously the case without electromagnetic field is trivially included here when $\mathbf{F} = 0$. Assuming that \mathbf{P} also has the structure

(8), then \mathbf{E} will share this same structure, so that \mathbf{T} , \mathbf{P} and \mathbf{E} will all be invariant under the action of the $\text{SO}(3)$ group.

Consider a general 2-form \mathbf{F} and demand that its energy-momentum tensor (10) has the structure (8). This imposes several restrictions on its components and one can easily prove that only two possibilities arise, depending on whether \mathbf{F} is null or not:

$$\mathbf{F} = \psi \mathbf{u} \wedge \mathbf{n} + \phi \boldsymbol{\omega}_\theta \wedge \boldsymbol{\omega}_\varphi = \Phi (\cos \eta \mathbf{u} \wedge \mathbf{n} + \sin \eta \boldsymbol{\omega}_\theta \wedge \boldsymbol{\omega}_\varphi), \quad (\text{non-null}) \quad (11)$$

$$\mathbf{F} = \Phi [\cos \eta (\mathbf{l} \wedge \boldsymbol{\omega}_\theta) + \sin \eta (\mathbf{l} \wedge \boldsymbol{\omega}_\varphi)] \quad (\text{null}) \quad (12)$$

where $\Phi = \Phi(u, R)$ and $\eta = \eta(u, R, \theta, \varphi)$, and \mathbf{l} is a future-pointing radial null vector field tangent to the (outgoing or ingoing) radial null geodesics. We also assume that \mathbf{l} is affinely parametrized. The corresponding energy-momentum tensors \mathbf{E} read

$$\mathbf{E} = \frac{\Phi^2}{8\pi} (\mathbf{u} \otimes \mathbf{u} - \mathbf{n} \otimes \mathbf{n} + \boldsymbol{\omega}_\theta \otimes \boldsymbol{\omega}_\theta + \boldsymbol{\omega}_\varphi \otimes \boldsymbol{\omega}_\varphi), \quad (\text{non-null}) \quad (13)$$

$$\mathbf{E} = \frac{\Phi^2}{4\pi} \mathbf{l} \otimes \mathbf{l}. \quad (\text{null}) \quad (14)$$

The previous restrictions are purely algebraic. In addition, the electromagnetic field \mathbf{F} must also satisfy the Maxwell equations

$$d\mathbf{F} = 0 \quad (15)$$

$$\delta\mathbf{F} = 4\pi\mathbf{J} \quad (16)$$

where δ is the co-differential (or divergence operator) and \mathbf{J} is the *electromagnetic current* 1-form. These equations impose further restrictions on the components of \mathbf{F} as follows:

a) *Non-null electromagnetic field.* Equations (15-16) applied to the electromagnetic field (11) lead to

$$d(R^2\phi) = 0 \quad \Rightarrow \quad \boxed{\phi = c/R^2} \quad (17)$$

$$\frac{-1}{4\pi R^2} [i(\mathbf{u})d(R^2\psi)\mathbf{n} + i(\mathbf{n})d(R^2\psi)\mathbf{u}] = \mathbf{J} \quad (18)$$

where c is a constant¹, and $i()$ indicates inner contraction. This implies that η (and therefore ϕ and ψ) can only depend on u and R .

¹The possibility $c \neq 0$ will be constrained later in this section and will turn out to be impossible in our models as proved in section 5.

Notice that \mathbf{J} can be split into two components, one along \mathbf{u} and the other along \mathbf{n} . Thus, an observer whose world lines are tangent to \mathbf{u} will measure a *charge density* given by

$$\varrho = -J^\alpha u_\alpha = \frac{i(\mathbf{n})d(R^2\psi)}{4\pi R^2}$$

and a *conduction current* [3]

$$\mathbf{j} \equiv \mathbf{J} - \varrho \mathbf{u} = -\frac{i(\mathbf{u})d(R^2\psi)}{4\pi R^2} \mathbf{n} . \quad (19)$$

The total charge q within a 2-sphere S defined by constant values of u and R is [40]:

$$q = \frac{1}{4\pi} \int_S * \mathbf{F} \quad \Longrightarrow \quad q = \psi R^2 \quad (20)$$

where $*\mathbf{F}$ is the Hodge dual of \mathbf{F} .

b) *Null electromagnetic field*: The Maxwell equations (15-16) applied to the electromagnetic field (12) provide now

$$i(\mathbf{k})d(R\Phi \cos \eta) = 0 \quad , \quad (21)$$

$$i(\mathbf{k})d(R\Phi \sin \eta) = 0 \quad , \quad (22)$$

$$i(\boldsymbol{\omega}_\varphi)d(\cos \eta) \sin \theta - i(\boldsymbol{\omega}_\theta)d(\sin \eta \sin \theta) = 0 \quad , \quad (23)$$

$$\frac{\Phi}{4\pi} [i(\boldsymbol{\omega}_\theta)d(\cos \eta \sin \theta) \csc \theta - i(\boldsymbol{\omega}_\varphi)d(\sin \eta)] \mathbf{l} = \mathbf{J} \quad , \quad (24)$$

where $\mathbf{k} \cdot \mathbf{k} = 0$, $\mathbf{l} \cdot \mathbf{k} = -1$ and, in order to write \mathbf{J} in the form (24), we have used equations (21-22). From here we infer that the electromagnetic 4-current 24 can only be null or zero and that the charge in any 2-sphere must be zero: $q = 0$, $\forall(u, R)$. Observe that if $\sin \eta \neq 0$ we can write (24), using (23), as

$$\mathbf{J} = -\frac{\Phi}{4\pi} \csc \eta \, i(\boldsymbol{\omega}_\theta)d\eta \mathbf{l} \quad (25)$$

while if $\sin \eta = 0$ then (24) becomes

$$\mathbf{J} = \frac{\Phi}{4\pi R} \cot \theta \mathbf{l} . \quad (26)$$

The case of physical relevance is that with no sources for the null electromagnetic field or, in other words, with $\mathbf{J} = 0$. From (25-26), we see that this is only possible if $i(\boldsymbol{\omega}_\theta)d\eta = 0$ and $\sin \eta \neq 0$. However, (23) implies then that $\eta_{,\varphi} = \cos \theta$, which is clearly incompatible with the previous formulas. Therefore, we conclude that *there cannot exist a sourceless null electromagnetic field whose energy-momentum tensor*

is $SO(3)$ invariant. If these sources existed the associated electromagnetic 4-current should be a null vector, a possibility which has a questionable physical interpretation (see e.g. [9], [11] and references therein). Thus, let us remark the well-known result that the only electromagnetic field invariant under the action of the group $SO(3)$ is the non-null one defined by (11,17-18).

As is well known, the tensor (13) obeys the dominant energy conditions by itself, and we have assumed that the whole energy-momentum tensor (9) so does. Nevertheless, this does not imply that \mathbf{P} must satisfy the DEC. However, as we wish to avoid negative energy densities or space-like energy fluxes associated with \mathbf{P} , in what follows we are going to demand the fulfillment of the dominant energy conditions for this part too. Taking into account (13), a straightforward calculation proves that conditions (3) and (5) remain the same while the inequalities (4) and (6) must be replaced by the stronger requirements:

$$\Phi^2 \leq \frac{2}{R^2}(m_{,R} - R\chi\beta_{,R}), \quad (27)$$

$$2\Phi^2 \leq \lambda + P_2, \quad (28)$$

$$0 \leq \lambda - P_2. \quad (29)$$

At this stage, we can already deduce important consequences from these inequalities. For instance, the energy condition (27) has an important physical interpretation. Using that $\Phi^2 = \phi^2 + \psi^2$, the definition (20) of q , and the result (17) for ϕ , we can rewrite (27) as:

$$q^2 + c^2 \leq 2R^2(m_{,R} - R\chi\beta_{,R}). \quad (30)$$

Recall that q is a function of u and R . Thus, if the space-time is given (that is, if we know $m(u, R)$ and $\beta(u, R)$ explicitly), then this relation can be interpreted as providing a maximum value to the charge enclosed in every 2-sphere.

If we now demand also the absence of curvature singularities at $R = 0$ (see (2)), from (30) we get that, in a neighborhood of $R = 0$,

$$q^2 + c^2 \leq 2[3m_3(u) - 2\beta_2(u)]R^4 + \text{higher order terms}. \quad (31)$$

Hence, as c is a constant and the second member of the inequality is zero for $R = 0$, we get that $c = 0$ and $q(u, 0) = 0$, and by using (17), we arrive at the following

Proposition 3.1 *If a spherically symmetric space-time has no curvature singularities at $R = 0$ and the energy condition (27) is fulfilled, then*

$$\phi = 0 \quad , \quad q(u, R \rightarrow 0) = 0. \quad (32)$$

In other words, the radial observers will not measure any magnetic field (since its value is proportional to ϕ) and there is no charge contained in $R = 0$. In this case, (30) can be expressed as²

$$\lim_{R \rightarrow 0} \frac{|q(u, R)|}{R^2} \leq \sqrt{6m_3(u) - 4\beta_2(u)} .$$

As a matter of fact, the requirement of absence of singularities at $R = 0$ is not necessary and, as is clear from the previous reasonings, one only needs the fulfillment of

$$\lim_{R \rightarrow 0} R^2(m_{,R} - R\chi\beta_{,R}) = 0. \quad (33)$$

Thus, there is a stronger version of proposition 3.1 where the demand of absence of singularities is replaced by the milder condition (33), in which case there may be a curvature singularity. As a particular case of this, suppose that we can carry out Taylor's expansions of m and β in a neighborhood of $R = 0$:

$$\begin{aligned} m(u, R) &= m_0(u) + m_1(u)R + m_2(u)R^2 + O(R^3), \\ \beta(u, R) &= \beta_0(u) + \beta_1(u)R + \beta_2(u)R^2 + O(R^3), \end{aligned}$$

then

$$\lim_{R \rightarrow 0} \frac{|q(u, R)|}{R} \leq \sqrt{2m_1(u) + 4m_0(u)\beta_1(u)} \quad (34)$$

where, as before, the righthand side is real due to (27).

4 The Vaidya-Reissner-Nordström spacetime

As stated in the introduction, the spherically symmetric space-time $\bar{\mathcal{V}}$ will be taken as the Vaidya-Reissner-Nordström (V-RN) solution [52], whose line-element is given by (1) with $2m(u, R) = 2M(u) - Q^2/R$ and $\beta(u, R) = 0$, that is

$$ds^2 = - \left(1 - \frac{2M(u)}{R} + \frac{Q^2}{R^2} \right) du^2 + 2\varepsilon du dR + R^2 d\Omega^2, \quad (35)$$

where $M(u)$ depends only on u and Q is a constant. Among the most remarkable known particular cases of (35) we can cite: if $M(u)$ is constant and $Q \neq 0$, then it coincides with Reissner-Nordström's (RN's) solution, including its particular cases of Schwarzschild's solution ($Q = 0$) and flat Minkowski's spacetime ($M = Q = 0$),

²(27) guarantees that the square root is real.

all of them very well-known and studied (see, e.g., [22]); if $M(u)$ is not constant and $Q = 0$, then (35) reduces to Vaidya's radiating solution [52], whose structure and possible extensions have been treated in [33, 26, 17], including the possibility of producing naked singularities (see, e.g., [29].) A general study of this solution was also carried out in [51].

Whenever $Q \neq 0$, the 2-form

$$\mathbf{F} = \frac{Q}{R^2} \mathbf{u} \wedge \mathbf{n} . \quad (36)$$

satisfies Maxwell's equations in the metric (35) and thus Q is interpreted as a charge and (36) as an electromagnetic field present in the spacetime. The energy-momentum tensor \mathbf{T} associated with (35) is given by

$$\mathbf{T} = \frac{\varepsilon}{4\pi R^2} \frac{dM(u)}{du} \mathbf{l} \otimes \mathbf{l} + \mathbf{E} \quad (37)$$

where the electromagnetic part is given by (13) with $\Phi = \psi = Q/R^2$, clearly describing a pure spherically symmetric non-null electrostatic field of total charge Q , and $\mathbf{l} = -du$ is the future-pointing radial null vector tangent to the radial null geodesics with $u = \text{constant}$. Thus, the non-electromagnetic part of the energy-momentum tensor describes a kind of incoherent radially outgoing (respectively ingoing) radiation filling the exterior region of the spherical body whose total mass-energy, given by $M(u)$, decreases with the retarded time u for $\varepsilon = -1$ (resp. increases if $\varepsilon = +1$) as a result of the isotropic radial emission (resp. absorption). By analogy with (14), one might think that this kind of radial radiation can be in fact electromagnetic radiation. However, we have proved in section 3 that this is not possible, since there can be no null electromagnetic field without sources satisfying the Maxwell equations (21-24) in this space-time whose stress-energy tensor matches the form of $\mathbf{T} - \mathbf{E}$ in (37), as should be expected [9].

Observe that the energy conditions for the radiative part of the energy-momentum tensor hold if

$$\varepsilon \frac{dM(u)}{du} \geq 0, \quad (38)$$

and that the V-RN spacetime is an example without the properties considered at the end of the previous section, because it has a strong curvature singularity at $R = 0$, and the milder condition (33) is also clearly violated whenever $Q \neq 0$. The global structure of this space-time if $Q \neq 0$ was analyzed in [17]. There appear three cases

depending on whether or not there exists u_1 such that $M^2(u_1) = Q^2$ and, if not, on whether $M^2(u)$ is greater than Q^2 or not. We refer the reader to [17, 51] for an account of the particular features in each case and for the corresponding Penrose diagrams of the V-RN spacetimes.

5 The gravitational-electromagnetic matching

In order to match a general spherically symmetric space-time \mathcal{V} with the Vaidya-Reissner-Nordström space-time $\bar{\mathcal{V}}$ ³ across a time-like hypersurface Σ , we are going to impose the gravitational *and* electromagnetic junction conditions. Specifically, we demand that the Einstein equations hold, in the distributional sense, in the whole space-time in such a way that no infinite jumps are allowed for the stress-energy tensor (thus, the possibility of a thin shell of matter on Σ is not considered). On the other hand, the Maxwell equations are also assumed to be valid, in a distributional sense, in the whole spacetime but allowing in this case for the existence of a charge surface density so that the electromagnetic 4-current may have an infinite jump at Σ .

The practical technique to perform the matching is to consider \mathcal{V} and $\bar{\mathcal{V}}$ as divided, each one, by the candidate matching hypersurfaces σ (respectively, $\bar{\sigma}$) into two parts –say 1 and 2 for \mathcal{V} and $\bar{1}$ and $\bar{2}$ for $\bar{\mathcal{V}}$ –. If σ and $\bar{\sigma}$ are diffeomorphic, one can identify corresponding points in them and then try to perform the gluing which, in principle, can be done in four different ways, namely, $1 - \bar{2}$; $1 - \bar{1}$; $2 - \bar{2}$; $2 - \bar{1}$. We can select one of them by choosing the relative sign of the normal vectors to the matching hypersurfaces σ , $\bar{\sigma}$, see [18] for details. Notice that, as these normal vectors are spacelike then we can always choose the tetrad such that \mathbf{n} and $\bar{\mathbf{n}}$ coincide, on σ and $\bar{\sigma}$ respectively, with them. Thus, from now on the unit normal vectors will also be denoted by \mathbf{n} and $\bar{\mathbf{n}}$.

Now, once σ and $\bar{\sigma}$ have been identified in the new glued spacetime there is no need to distinguish them and they will be simply termed as Σ if there is no confusion. If Σ is timelike and preserves the spherical symmetry then it can be described by intrinsic coordinates $\{\xi, \vartheta, \varphi\}$ where ξ is a timelike coordinate and where $\{u(\xi), R(\xi), \theta = \vartheta, \phi = \varphi\}$ and $\{\bar{u}(\xi), \bar{R}(\xi), \bar{\theta} = \vartheta, \bar{\phi} = \varphi\}$ are the parametric representations

³We will set an overbar on all variables, parameters and functions of the space-time $\bar{\mathcal{V}}$ to distinguish them from the corresponding objects of the general space-time \mathcal{V} .

of σ in \mathcal{V} and of $\bar{\sigma}$ in $\bar{\mathcal{V}}$, respectively. Before going any further, and as pointed out in [12, 36], one has to specify how the tangent planes at every point $p \in \sigma$ and at its corresponding point $\bar{p} \in \bar{\sigma}$ must be identified in order to construct a well-defined geometry in the whole glued space-time. To do that, we proceed as follows: firstly we identify the vector fields associated to the angular variables, $\partial/\partial\theta$ with $\partial/\partial\bar{\theta}$, and $\partial/\partial\varphi$ with $\partial/\partial\bar{\varphi}$, at Σ ; secondly we consider the two vector fields defined on σ and $\bar{\sigma}$ respectively by

$$\left(\frac{\partial R(\xi)}{\partial \xi} \frac{\partial}{\partial R} + \frac{\partial u(\xi)}{\partial \xi} \frac{\partial}{\partial u} \right) \Big|_{\sigma} \quad \text{and} \quad \left(\frac{\partial \bar{R}(\xi)}{\partial \xi} \frac{\partial}{\partial \bar{R}} + \frac{\partial \bar{u}(\xi)}{\partial \xi} \frac{\partial}{\partial \bar{u}} \right) \Big|_{\bar{\sigma}} \quad (39)$$

and identify them since they both represent $\partial/\partial\xi$ at Σ ; thirdly, take the unit normal vectors \mathbf{n} (resp., $\bar{\mathbf{n}}$) for σ (resp., $\bar{\sigma}$) defined, except for a sign, by being orthogonal to the three previous vectors at Σ , and choose these signs in such a way that every curve crossing Σ through a point $p \equiv \bar{p}$ must have a unique well-defined tangent vector there [18]; and fourthly, by doing so, the relative sign ϵ_n ($\epsilon_n^2 = 1$) of the normal unit vectors has been fixed so that we can identify them thereby achieving a complete identification of the two tangent planes. As a by-product, this process determines an orientation and an arrow of time for the glued space-time if these concepts are well defined or fixed in some way in \mathcal{V} and $\bar{\mathcal{V}}$.

At this stage, we impose the Darmois gravitational junction conditions [13][18], which are the best suited for our purposes, requiring that the first and second fundamental forms of Σ be identical when computed from either \mathcal{V} or $\bar{\mathcal{V}}$. After the appropriate calculations, these matching conditions can be written in two (equivalent) complete sets, depending on the sign ϵ_n , as follows

Case $\varepsilon\bar{\varepsilon} = \epsilon_n$

$$R \stackrel{\Sigma}{=} \bar{R}, \quad (40)$$

$$\varepsilon\dot{u}e^{2\beta} \stackrel{\Sigma}{=} \bar{\varepsilon}\dot{\bar{u}}, \quad (41)$$

$$m \stackrel{\Sigma}{=} \bar{M} - \frac{\bar{Q}^2}{2\bar{R}}, \quad (42)$$

$$-\varepsilon \left[\left(1 - \frac{2m}{R} \right) \beta_{,R} - \frac{m_{,R}}{2R} \right] e^{2\beta} \dot{u} + \beta_{,R} \dot{R} \stackrel{\Sigma}{=} \bar{\varepsilon} \frac{\bar{Q}^2}{4\bar{R}^3} \dot{\bar{u}}. \quad (43)$$

Case $\varepsilon\bar{\varepsilon} = -\epsilon_n$

$$R \stackrel{\Sigma}{=} \bar{R}, \quad (44)$$

$$\varepsilon\chi e^{2\beta} \dot{u} - 2\dot{R} \stackrel{\Sigma}{=} -\bar{\varepsilon}\bar{\chi} \dot{\bar{u}}, \quad (45)$$

$$m \stackrel{\Sigma}{=} \bar{M} - \frac{\bar{Q}^2}{2\bar{R}}, \quad (46)$$

$$\varepsilon \left[\chi \left(\chi_{\beta,R} - \frac{m_{,R}}{2R} \right) + \varepsilon \frac{m_{,u} e^{-2\beta}}{R} \right] e^{2\beta} \dot{u} + \left(\frac{m_{,R}}{R} - \chi_{\beta,R} \right) \dot{R} \stackrel{\Sigma}{=} \bar{\varepsilon} \frac{\bar{Q}^2}{4\bar{R}^3} \bar{\chi} \dot{\bar{u}}. \quad (47)$$

Here $\stackrel{\Sigma}{=}$ means that we have to compute both sides of the equality at Σ , and the overdots stand for derivatives with respect to ξ .

In practice, we can understand the equivalence of the two sets of matching conditions by noting that they are suitable to describe the prolongation, when crossing Σ , of a given radial null geodesic with $u = \text{constant}$ by either a radial null geodesic with $\bar{u} = \text{constant}$ (if $\varepsilon\bar{\varepsilon} = \epsilon_n$), or not (if $\varepsilon\bar{\varepsilon} = -\epsilon_n$), see [51] for more details.

The above takes care of the *gravitational* matching, but we still require an appropriate junction of the electromagnetic fields. To that end, we present now the *general* matching conditions for the electromagnetic field (for arbitrary space-times) and then we will first particularize them to the spherically symmetric case and in a latter step to the case in which one of the matching space-times ($\bar{\mathcal{V}}$) is the V-RN solution. Let \mathbf{F} and $\bar{\mathbf{F}}$ be the respective electromagnetic 2-forms at \mathcal{V} and $\bar{\mathcal{V}}$, so that they satisfy the Maxwell equations

$$d\mathbf{F} = 0 \quad \delta\mathbf{F} = 4\pi\mathbf{J} \quad (48)$$

$$d\bar{\mathbf{F}} = 0 \quad \delta\bar{\mathbf{F}} = 4\pi\bar{\mathbf{J}} \quad (49)$$

on \mathcal{V} and $\bar{\mathcal{V}}$, respectively. The whole electromagnetic field is defined to be

$$\mathcal{F} = \mathbf{F}(1 - \theta_\Sigma) + \bar{\mathbf{F}}\theta_\Sigma \quad (50)$$

where the Heaviside theta function θ_Σ is defined by $\theta_\Sigma|_{\mathcal{V}} = 0$ and $\theta_\Sigma|_{\bar{\mathcal{V}}} = 1$. Thus, it is immediate to get

$$d\mathcal{F} = (\mathbf{F} \wedge \mathbf{n} - \bar{\mathbf{F}} \wedge \bar{\mathbf{n}}) \delta_\Sigma \quad (51)$$

$$\delta\mathcal{F} = 4\pi[\mathbf{J}(1 - \theta_\Sigma) + \bar{\mathbf{J}}\theta_\Sigma] + [i(\mathbf{n})\mathbf{F} - i(\bar{\mathbf{n}})\bar{\mathbf{F}}] \delta_\Sigma \quad (52)$$

where \wedge is the exterior product and δ_Σ is the normalized Dirac delta with support on Σ , see e.g. [12, 36], defined by $d\theta_\Sigma = \mathbf{n} \delta_\Sigma$. It follows from (51,52) that the fulfillment of the Maxwell equations for \mathcal{F} in a distributional sense provides the sought-after *electromagnetic* junction conditions:

$$\mathbf{F} \wedge \mathbf{n} \stackrel{\Sigma}{=} \bar{\mathbf{F}} \wedge \bar{\mathbf{n}} \quad (53)$$

$$\mathcal{J} \equiv \mathbf{J}(1 - \theta_\Sigma) + \bar{\mathbf{J}}\theta_\Sigma + \mathcal{K}\delta_\Sigma, \quad (54)$$

where \mathcal{J} is the distributional 4-current and

$$\mathcal{K} \stackrel{\Sigma}{=} \frac{1}{4\pi} [i(\mathbf{n})\mathbf{F} - i(\bar{\mathbf{n}})\bar{\mathbf{F}}] \quad (55)$$

is the *surface current density* 4-vector (obviously defined only on Σ).

If we do not wish to allow the total 4-current to have infinite jumps at Σ , then from (54) and (55) we must impose

$$i(\mathbf{n})\mathbf{F} \stackrel{\Sigma}{=} i(\bar{\mathbf{n}})\bar{\mathbf{F}} \quad (56)$$

so that, taking into account (53) we derive $\mathbf{F} \stackrel{\Sigma}{=} \bar{\mathbf{F}}$ in this case.

Let us now consider the particular case we are interested in: spherical symmetry. From conditions (53) one immediately infers that a non-null electromagnetic field (resp., a null one) can only be matched with another non-null field (resp., null). But as we proved before, null electromagnetic fields are not compatible with spherically symmetric energy-momentum tensors, so that only the non-null case must be considered for our purposes⁴. Hence, the electromagnetic fields take the form (11)

$$\mathbf{F} = \psi \mathbf{u} \wedge \mathbf{n} + \phi \omega_\theta \wedge \omega_\varphi \quad \text{and} \quad \bar{\mathbf{F}} = \bar{\psi} \bar{\mathbf{u}} \wedge \bar{\mathbf{n}} + \bar{\phi} \bar{\omega}_\theta \wedge \bar{\omega}_\varphi$$

in \mathcal{V} and $\bar{\mathcal{V}}$, respectively, so that the matching conditions (53,54) imply

$$\phi \stackrel{\Sigma}{=} \bar{\phi} \quad (57)$$

$$\mathcal{K} = \frac{1}{4\pi} (\bar{\psi} - \psi) \mathbf{u} . \quad (58)$$

Let us finally particularize to the matching with the V-RN solution. Condition (57) applied to the electromagnetic fields (11) and (36) becomes

$$\phi \stackrel{\Sigma}{=} 0 \quad (59)$$

which, together with (17), leads to vanishing of ϕ on the entire \mathcal{V} , and not just on Σ :

$$\phi = 0.$$

This result means that the radially moving observers cannot measure any magnetic fields on \mathcal{V} . Similarly, taking into account that the “charge function” on \mathcal{V} is given

⁴If one does not demand the invariance of the null field energy-momentum tensor and computes the junction conditions (53,54) for the null-null case one easily gets $\mathcal{K} = 0$ and $\mathbf{F} \stackrel{\Sigma}{=} \bar{\mathbf{F}}$, see [51].

by $q(u, R) = \psi R^2$ [see (20)], we can define the *total charge* $Q \stackrel{\Sigma}{\equiv} \psi R^2$. Using now (36), we can express \mathcal{K} of (58) as

$$\mathcal{K} \stackrel{\Sigma}{\equiv} \frac{1}{4\pi R^2} (\bar{Q} - Q) \mathbf{u}$$

so that, using (20), we are led to define the *total surface charge* in Σ by

$$Q_{\Sigma} \equiv \bar{Q} - Q \stackrel{\Sigma}{\equiv} \bar{Q} - \psi R^2.$$

This is a very natural result and implies that the sum of the total charge Q contained in \mathcal{V} plus the total surface charge Q_{Σ} must be constant ($= \bar{Q}$). Of course, the presence of infinite jumps on the electromagnetic 4-current is a mathematical idealization that can, in many cases, simplify the complexity of describing some models with a large concentration of charge at Σ . However, the absence of infinite jumps in the 4-current is more realistic physically and, in fact, it is in agreement with the experimental observations [28]. Therefore, if in particular we do not allow for infinite jumps in the 4-current or, in other words, if there is not a surface charge we obtain

$$Q \stackrel{\Sigma}{\equiv} \psi R^2 \stackrel{\Sigma}{=} \bar{Q}. \quad (60)$$

Then the total charge Q must also be constant.

In the rest of this section, we are going to focus on this physically more realistic case when (60) is fulfilled. Then, the complete set of matching equations (40)-(43), (59) and (60) (or alternatively (44)-(47), (59) and (60)) are the necessary and sufficient conditions for the matching of a general spherically symmetric metric with the V-RN solution through a general spherically symmetric timelike hypersurface, and they provide relations between relevant quantities at both sides of Σ . Nonetheless, it is remarkable that a linear combination of (40), (60) and (43) leads to

$$\varepsilon \left[\left(1 - \frac{2m}{R} \right) \beta_{,R} - \frac{m_{,R}}{2R} + \frac{Q^2}{4R^3} \right] \dot{u} - \beta_{,R} e^{-2\beta} \dot{R} \stackrel{\Sigma}{=} 0, \quad (61)$$

or alternatively, for the other case with $\varepsilon \bar{\varepsilon} = -\epsilon_n$, one similarly finds

$$\varepsilon \left[\chi \left(\chi \beta_{,R} - \frac{m_{,R}}{2R} \right) + \varepsilon \frac{m_{,u} e^{-2\beta}}{R} + \frac{Q^2 \chi}{4R^3} \right] \dot{u} + \left(\frac{m_{,R}}{R} - \chi \beta_{,R} - \frac{Q^2}{2R^3} \right) e^{-2\beta} \dot{R} \stackrel{\Sigma}{=} 0. \quad (62)$$

Clearly, these relations involve quantities of the space-time \mathcal{V} *only*, but *not* of $\bar{\mathcal{V}}$. This means that equation (61) (or (62)) is a necessary condition that the space-time

\mathcal{V} , by itself, must fulfill on Σ in order to be matchable to a V-RN solution. The physical meaning of (61) (or (62)) generalizes the results for standard models in which normal pressures must vanish on Σ when matching to a vacuum, as will be explained in section 6 in detail.

Given the previous remarks, one can now derive much more information from the matching equations. To extract this information, one has to follow paths which depend on the known data of the problem under consideration. A physically very interesting case arises when $m(u, R)$ and $\beta(u, R)$ are given, so that the spacetime \mathcal{V} is completely known, and we want to ascertain whether or not \mathcal{V} is *matchable* to a V-RN solution, if so *where*, and finally *to which particular* V-RN metric, that is, for which particular $\bar{M}(\bar{u})$ and \bar{Q} . In order to solve this problem, first of all we treat $\bar{Q}(\stackrel{\Sigma}{=} Q)$ either as known or as a parameter on which the final results will depend. This is a key point. Then, we proceed as follows: for the $\varepsilon\bar{\varepsilon} = \epsilon_n$ case (resp., $\varepsilon\bar{\varepsilon} = -\epsilon_n$), the equation (61) (resp., (62)), can be considered as an ordinary differential equation for $R(u)$, with the form $dR/du = F(u, R; \bar{Q})$, so that if $m(u, R)$ and $\beta(u, R)$ are such that $F(u, R; \bar{Q})$ satisfies Lipschitz's conditions we can find the solution $R(u; c_1, \bar{Q})$, where c_1 is an integration constant. If $R(u; c_1, \bar{Q})$ defines a time-like Σ on \mathcal{V} , then using (42) we can determine $\bar{M} \stackrel{\Sigma}{=} \bar{M}(u; c_1, \bar{Q})$. Now, by integrating (41) for the first case –or (45) for the second– we get $\bar{u}(u; c_1, \bar{Q})$, except for a new additive constant c_2 ,⁵ while from (40) (or, respectively, from (44)) we get $\bar{R}(u; c_1, \bar{Q})$. These two functions define the hypersurface Σ in the space-time $\bar{\mathcal{V}}$. Finally, due to the fact that $\bar{M}(\bar{u})$ is a function of only \bar{u} , by combining $\bar{M}(u; c_1, \bar{Q})$ with $\bar{u}(u; c_1, \bar{Q})$ we get $\bar{M}(\bar{u}; c_1, \bar{Q})$ and the problem is completely solved.

The solutions $R(u; c_1, \bar{Q})$ give a *two-parameter* family of matching hypersurfaces through which the space-time \mathcal{V} is matchable with a V-RN solution. The different values for c_1 (and of course \bar{Q}) will simply lead to the different particular V-RN space-times which match at the various Σ 's, providing explicitly $\bar{M}(\bar{u})$ (and \bar{Q}). In order to interpret the physical differences between these Σ 's, consider their intersection with a given null hypersurface $u = u_0 = \text{constant}$. The values of $R \stackrel{\Sigma}{=} \bar{R}$ and \bar{M} at these intersections are denoted by $R_0 = R(u_0; c_1, \bar{Q})$ and $\bar{M}_0 = \bar{M}(u_0; c_1, \bar{Q})$, so that they depend on the value of c_1 , that is, on the particular Σ . Eliminating c_1 from \bar{M}_0 and R_0 we obtain $\bar{M}_0 = \bar{M}_0(u_0, R_0, \bar{Q})$ which gives, at the given u_0 , the

⁵The specific value of c_2 is irrelevant since it only provides an origin for the *times* u and \bar{u} , and therefore we choose, for the sake of simplicity, $c_2 = 0$.

value of \bar{M}_0 as a function of R_0 . Use of (42) [or (46)] leads to the explicit expression

$$\bar{M}_0(u_0, R_0, \bar{Q}) = m(u_0, R_0) + \frac{\bar{Q}^2}{2R_0}.$$

Differentiating here with respect to R_0 and taking into account the energy condition (27) with $\bar{Q} \stackrel{\Sigma}{=} Q$ we get

$$\frac{d\bar{M}_0}{dR_0} = m_{,R}(u_0, R_0) - \frac{\bar{Q}^2}{2R_0^2} \geq R_0 (\chi\beta_{,R})|_{u_0, R_0}.$$

Finally, if the hypersurface at u_0 is not at a region with trapped 2-spheres, that is to say, $\chi(u_0, R_0) \geq 0$, then from the energy condition (3) we finally infer

$$\frac{d\bar{M}_0}{dR_0} \geq 0.$$

In other words, given a space-time \mathcal{V} take the 2-spheres (which belong to a matching hypersurface Σ) defined by $\Sigma \cap \{u = u_0\} \cap \{\chi \geq 0\}$. These will have an area proportional to R_0^2 and contain a total mass M_0 . The previous result tells us that *smaller* such 2-spheres will contain non-bigger masses.

6 Implications on the matter content of \mathcal{V}

In the previous sections we have not interpreted the matter contents of the space-time \mathcal{V} apart from assuming that it may contain an electromagnetic field. Our purpose in this section is to investigate some of the different interpretations for the non-electromagnetic matter content and to elucidate the restrictions imposed by the matching conditions, found in the previous section, on the electromagnetic field as well as on the non-electromagnetic part of the energy-momentum tensor.

Let us start with the electromagnetic field in \mathcal{V} , given by (11) with (18) and

$$\psi \stackrel{\Sigma}{=} \bar{Q}/R^2, \quad \phi = 0.$$

Taking the derivative of the first of these equations along Σ we get, on using (18)

$$i(\mathbf{u})d(R^2\psi) \stackrel{\Sigma}{=} 0 \implies \mathbf{J} \stackrel{\Sigma}{=} \frac{-1}{4\pi R^2} i(\mathbf{n})d(R^2\psi)\mathbf{u}. \quad (63)$$

This tells us that \mathbf{J} is a time-like vector on Σ , furthermore comoving with Σ at Σ . Outside Σ , though, the form of \mathbf{J} is unrestricted, and in general it has two

components, one along the direction of the time-like \mathbf{u} , which is the convection current with respect to \mathbf{u} (assuming that \mathbf{u} is chosen to be the 4-velocity of the charged fluid), and another in the direction of \mathbf{n} , representing the corresponding conduction current with respect to \mathbf{u} , which *necessarily* vanishes at Σ . Therefore, any function ψ that satisfies (60) provides us with a regular electromagnetic field valid for \mathcal{V} .⁶

Now we can ask ourselves the physically relevant question of whether the electromagnetic field can be chosen only with convention, or only with conduction, current. This is clearly equivalent, according to (18), to finding out the causal character of the one-form $d(R^2\psi)$, because the sign of $\mathbf{J} \cdot \mathbf{J}$ is opposite to that of $d(R^2\psi) \cdot d(R^2\psi)$. Thus \mathbf{J} is time-like (resp. space-like) and thus can be aligned with \mathbf{u} (resp. \mathbf{n}) if $d(R^2\psi)$ is spacelike (resp. timelike). Observe that in the second case $\mathbf{J} \stackrel{\Sigma}{=} 0$ necessarily.

Finally, the energy-momentum tensor \mathbf{E} associated with the non-null electromagnetic field in \mathcal{V} is now (13) with $\phi = 0$:

$$\mathbf{E} = \frac{\psi^2}{8\pi}(\mathbf{u} \otimes \mathbf{u} - \mathbf{n} \otimes \mathbf{n} + \boldsymbol{\omega}_\theta \otimes \boldsymbol{\omega}_\theta + \boldsymbol{\omega}_\varphi \otimes \boldsymbol{\omega}_\varphi). \quad (64)$$

Let us now pass to the rest of the matter content. Hitherto, we only know that its energy-momentum tensor, $\mathbf{P} = \mathbf{T} - \mathbf{E}$, takes a form of type (8). As is known, see e.g. [48], the energy-momentum tensors for different *physical types* of matter distributions can, in fact, have precisely the same components. Therefore \mathbf{P} is opened to different interpretations, depending on the physical processes we want to describe.

One of the simplest interpretations that we can put forward, containing the minimum degrees of freedom, is the case of a *charged non-perfect fluid embedded in a radially directed null radiation*. The 4-velocity is taken to be \mathbf{u} so that the fluid is comoving with Σ by construction. The total energy-momentum tensor for this case is then written as

$$\mathbf{T} = (\mu + p) \mathbf{u} \otimes \mathbf{u} + p \mathbf{g} + \Omega^2 \ell \otimes \ell + \boldsymbol{\Pi} + \mathbf{E}, \quad (65)$$

where μ is the energy density of the fluid, p its isotropic pressure, $\boldsymbol{\Pi}$ its anisotropic pressure tensor (which is traceless and orthogonal to \mathbf{u}), Ω^2 is the null radiation

⁶If we allowed for infinite jumps on \mathbf{J} the problem would be *easier* since there would be no constraints on ψ , not even on the matching hypersurface, so that any function ψ would provide a regular electromagnetic field valid for \mathcal{V} .

energy density, $\ell = \mathbf{u} + s \mathbf{n}$ ($s^2 = 1$) is a radially directed null vector field, and \mathbf{E} is given in (64).

Obviously, the form (65) of the energy-momentum tensor is not unique and another physically relevant possible interpretation, also having the minimum degrees of freedom, is as a *charged non-perfect fluid with heat conduction*. In this case we write

$$\mathbf{T} = (\tilde{\mu} + \tilde{p})\mathbf{u} \otimes \mathbf{u} + \tilde{p}\mathbf{g} + \mathbf{h} \otimes \mathbf{u} + \mathbf{u} \otimes \mathbf{h} + \tilde{\mathbf{\Pi}} + \mathbf{E}, \quad (66)$$

where $\mathbf{h} \propto \mathbf{n}$ is the heat flux vector and $\tilde{\mu}$, \tilde{p} and $\tilde{\mathbf{\Pi}}$ have the usual interpretation. Of course, (65,66) are not the only possible cases and many other physically realistic decompositions can be written according to the particular situation.

In any case, and following the philosophy proposed in [16], for every given $m(u, R)$, $\beta(u, R)$ and $\psi(u, R)$ such that (60) is satisfied and a timelike Σ exists, the velocity vector field \mathbf{u} is fixed only on Σ and, provided that it is time-like and orthogonal to the 2-spheres $\{u, R\} = \text{constants}$, we have freedom to choose it in the rest of \mathcal{V} . Once \mathbf{u} is chosen we can compute, by means of (18), the 4-current \mathbf{J} , and by using Einstein's equations, μ , p , Ω^2 and $\mathbf{\Pi}$ for (65), or $\tilde{\mu}$, \tilde{p} , \mathbf{h} and $\tilde{\mathbf{\Pi}}$ for (66), or the corresponding quantities for any other desired interpretation of \mathbf{P} .

It is known that some relations between the energy-momentum tensor at both sides of Σ follow from the matching of two space-times [25, 36]. They read

$$\mathbf{T}_{\mu\nu}\tau^\mu n^\nu \stackrel{\Sigma}{=} \bar{\mathbf{T}}_{\mu\nu}\bar{\tau}^\nu \bar{n}^\nu, \quad (67)$$

$$\mathbf{T}_{\mu\nu}n^\mu n^\nu \stackrel{\Sigma}{=} \bar{\mathbf{T}}_{\mu\nu}\bar{n}^\mu \bar{n}^\nu, \quad (68)$$

where τ^μ (equivalently $\bar{\tau}^\mu$) is *any* vector tangent to Σ . Therefore, for the first interpretation proposed in (65), the non-trivial relations deriving from (67,68) are

$$\ell \stackrel{\Sigma}{=} \mathbf{1}, \quad (69)$$

$$\Omega^2 \stackrel{\Sigma}{=} \frac{\varepsilon}{4\pi R^2} \frac{dM(u)}{du}, \quad (70)$$

$$p + \Omega^2 + \Pi_{\mu\nu}n^\mu n^\nu - \frac{\psi^2}{8\pi} \stackrel{\Sigma}{=} \frac{\varepsilon}{4\pi R^2} \frac{dM(u)}{du} - \frac{\bar{Q}^2}{8\pi \bar{R}^4}, \quad (71)$$

where we have used (65) and (37). Combining the last two equations and using (60) we get

$$p + \Pi_{\mu\nu}n^\mu n^\nu \stackrel{\Sigma}{=} 0, \quad (72)$$

which *does not involve* any quantity from the space-time $\bar{\mathcal{V}}$.

The equalities (60), (70) and (72) are the main physical equations which relate quantities at both sides of Σ . The equation (70) expresses the fact that the radiated energy density has to be continuous through the matching hypersurface. On the other hand, it is easy to check that (72) is strictly equivalent to the matching condition (61) (or to (62)), see [16]. Its physical interpretation is clear, (72) says that the total normal pressure has to vanish on the matching hypersurface. All in all, we can reformulate the results from this and the previous sections in the following way

Theorem 6.1 *Every spherically symmetric metric can be locally matched to a Vaidya-Reissner-Nordström solution across any time-like hypersurface with the properties: 1) the total normal pressure of the fluid in the decomposition (65) vanishes on it; and 2) the total charge enclosed in it is constant.*

This is a very satisfactory result, which generalizes previous results for the uncharged case, in particular those obtained by Misner & Sharp [39] and Bel & Hamoui [2] for the case with a Schwarzschild exterior and absence of radiation; and those of Fayos et al. [16] for the Vaidya's radiative solution. It is also a generalization of previous results obtained by Oliveira & Santos [42] for charged shear-free fluids, although this case is better adapted to the second interpretation given in (66). We can actually reformulate the previous theorem in this case as follows:

Theorem 6.2 *Every spherically symmetric metric can be locally matched to a Vaidya-Reissner-Nordström solution across any time-like hypersurface whose total normal pressure in the decomposition (66) equals the heat flux on it, and such that the total charge enclosed inside it is constant.*

6.1 Brief note on the existence of a surface charge on Σ

If we allowed for the existence of infinite jumps in the electromagnetic 4-current the situation would be slightly different, and for example condition (72) would not be valid. Instead, we would have, from (70) and (71):

$$p + \Pi_{\mu\nu} n^\mu n^\nu - \frac{Q^2}{8\pi R^4} \stackrel{\Sigma}{=} -\frac{\bar{Q}^2}{8\pi \bar{R}^4}.$$

This can be easily understood by noting that the normal tensions exerted by the electromagnetic fields on Σ , given by $-Q^2/(8\pi R^4)$ and $-\bar{Q}^2/(8\pi \bar{R}^4)$, do not compensate reciprocally. A non-zero “normal pressure exerted by the fluid” is thus

needed in order to satisfy the equality of *total* normal pressures as required by (68). Given this, we can generalize theorems 6.1 and 6.2 as

Theorem 6.3 *If infinite jumps in the electromagnetic 4-current are allowed, every spherically symmetric metric can be locally matched to the Vaidya-Reissner-Nordström solution across any time-like hypersurface Σ such that*

$$4\pi R^4(p + \Pi_{\mu\nu} n^\mu n^\nu) - Q^2 \stackrel{\Sigma}{=} \text{constant} \leq 0 .$$

Theorem 6.4 *If we allow for infinite jumps in the electromagnetic 4-current, every spherically symmetric metric can be locally matched to a Vaidya-Reissner-Nordström solution across any time-like hypersurface satisfying*

$$4\pi R^4(\tilde{p} + \tilde{\Pi}_{\mu\nu} n^\mu n^\nu - h_\alpha n^\alpha) - Q^2 \stackrel{\Sigma}{=} \text{constant} \leq 0 .$$

Clearly these theorems are less restrictive than the corresponding theorems in the absence of surface charge, since the time-like hypersurfaces satisfying the requirements in theorem 6.1 (or 6.2) will also satisfy the corresponding requirements in theorem 6.3 (or, respectively, 6.4), but not the other way round.

7 Other important physical consequences

In this section we are going to prove that the energy and matching conditions imply several important physical consequences, namely, (i) inequalities valid on the matching hypersurface, one of them involving only quantities of \mathcal{V} ; (ii) the non-spacelike character of the matching hypersurface under general conditions; and (iii) several limits on the total charge of the model.

To that end, we are going to exploit the energy conditions on \mathbf{P} , given by (3), (5), and (27-29) supplemented with the fact that $\phi = 0$ and, therefore, $\Phi^2 = \psi^2$. This group of conditions constrain the functions $m(u, R)$, $\beta(u, R)$ and $\psi(u, R)$ that we can choose for \mathcal{V} and, when specialized to Σ , give physical restrictions valid on the matching hypersurface. For instance, the matching conditions for the $\varepsilon\bar{\varepsilon} = \epsilon_n$ case⁷ are the relations (40)-(42) and (61). We can isolate $m_{,R}$ and $m_{,u}$ from them

⁷Obviously the $\varepsilon\bar{\varepsilon} = -\epsilon_n$ case can be treated similarly.

in terms of magnitudes from $\bar{\mathcal{V}}$

$$m_{,R} \stackrel{\Sigma}{=} 2\bar{R} \left((\bar{\chi} - \bar{\varepsilon}\bar{R}')\beta_{,R} + \frac{\bar{Q}^2}{4\bar{R}^3} \right) \quad (73)$$

$$\varepsilon m_{,u} \stackrel{\Sigma}{=} \bar{\varepsilon} e^{2\beta} \left(\bar{M}_{,\bar{u}} - 2\bar{R}\beta_{,R}(\bar{\chi} - \bar{\varepsilon}\bar{R}')\bar{R}' \right) \quad (74)$$

where $\bar{R}' \equiv \dot{\bar{R}}/\dot{\bar{u}}$. Combining these with (27) and (5) we arrive at

$$0 \stackrel{\Sigma}{\leq} \bar{R}\beta_{,R}(\bar{\chi} - 2\bar{\varepsilon}\bar{R}') \quad (75)$$

$$0 \leq \bar{\varepsilon}\bar{M}_{,\bar{u}} \stackrel{\Sigma}{\leq} \frac{1}{2}\bar{R}\beta_{,R}(\bar{\chi} - 2\bar{\varepsilon}\bar{R}')^2 \quad (76)$$

where use have been made of the energy condition (38). On the other hand, if we differentiate (42) on Σ and replace \dot{R}/\dot{u} with the value arising from (61) we find

$$\varepsilon m_{,u}\beta_{,R} \stackrel{\Sigma}{\geq} \frac{e^{2\beta}}{8R^6} (Q^2 - 2R^2 m_{,R})(Q^2 - 2R^2 m_{,R} + 4\chi\beta_{,R}R^3). \quad (77)$$

where again the fulfillment of (38) has been used. Unlike (75) and (76), this inequality (77) only relates quantities from \mathcal{V} on σ , and it will be important in the construction of models. On the other hand, inequalities (75), (76) and (77) use up all the reciprocal implications of the dominant energy condition at both sides of the matching hypersurface.

More interestingly, we can derive under which circumstances the matching hypersurface will be actually non-spacelike⁸. The condition for this is $\dot{u}(2\varepsilon\dot{R} - e^{2\beta}\chi\dot{u}) \leq 0$. We can consider two possibilities:

1. $\dot{u} = 0$, so that Σ is null. Then we get:

- Case $\varepsilon\bar{\varepsilon} = \epsilon_n$, from (61):

$$\beta_{,R} \stackrel{\Sigma}{=} 0$$

- Case $\varepsilon\bar{\varepsilon} = -\epsilon_n$, from (62):

$$\frac{m_{,R}}{R} - \chi\beta_{,R} - \frac{Q^2}{2R^3} \stackrel{\Sigma}{=} 0$$

2. $\dot{u} > 0$, so that

$$\varepsilon \frac{\dot{R}}{\dot{u}} \stackrel{\Sigma}{\leq} \frac{e^{2\beta}}{2} \chi. \quad (78)$$

Isolating \dot{R}/\dot{u} from (61) [or from (62) for the second case] and replacing it into (78) we get

⁸Here we will use that (61) and (62) are valid no matter what the character of the matching hypersurface is, as was shown in [37].

- Case $\varepsilon\bar{\varepsilon} = \epsilon_n$:

$$\left(\text{if } \beta_{,R} \stackrel{\Sigma}{\neq} 0 \right) \Rightarrow \frac{m_{,R}}{R} - \chi\beta_{,R} - \frac{Q^2}{2R^3} \stackrel{\Sigma}{\geq} 0 \quad (79)$$

- Case $\varepsilon\bar{\varepsilon} = -\epsilon_n$:

$$\left(\text{if } \frac{m_{,R}}{R} - \chi\beta_{,R} - \frac{Q^2}{2R^3} \stackrel{\Sigma}{\neq} 0 \right) \Rightarrow \chi^2\beta_{,R} + \epsilon \frac{2m_{,u} e^{-2\beta}}{R} \stackrel{\Sigma}{\geq} 0 \quad (80)$$

These relations are satisfied if (27) and (5) hold. Furthermore, $\beta_{,R} > 0$ and $\chi^2\beta_{,R} + \epsilon(2m_{,u} e^{-2\beta})/R > 0$, with the strict inequality, are part of the energy conditions for energy-momentum tensors of type I (see [22] for definitions). Therefore, we have

Proposition 7.1 *If the energy-momentum tensor in \mathcal{V} is of “type I” and its non-electromagnetic part satisfies the dominant energy conditions, and if no infinite jumps are allowed for the electromagnetic 4-current, then any matching hypersurface Σ with a Vaidya-Reissner-Nordström solution is necessarily non-spacelike.*

More specifically, assuming that (27) holds, it is easy to see that the matching hypersurface will generically be timelike, and it may be null only in the very particular case that $m_{,R}/R - \chi\beta_{,R} - Q^2/(2R^3) \stackrel{\Sigma}{\equiv} 0$ holds.

On the other hand, *type II* energy-momentum tensors are usually associated with zero rest mass fields (recall that, for instance, the Vaidya and V-RN solutions have a *type II* energy-momentum tensor). We can consider the question of whether or not a proposition equivalent to 7.1 (relating the energy conditions for \mathcal{V} with the non-spacelike nature of Σ) can be found when the energy-momentum tensor is *type II*. It is easy to see that such proposition *cannot exist in general*, since the matching conditions in the previous section show that, in the particular case of two matchable V-RN space-times, the nature of the matching hypersurface can be arbitrary independently of any other conditions.

Let us consider finally the important question of whether there are any upper limits on the total charge of the model. From proposition 2.2 evaluated on Σ and using (42) [or (46)] we immediately obtain

Proposition 7.2 *If $m(u, 0) \geq 0$ and the dominant energy conditions hold, then*

$$\bar{Q}^2 \stackrel{\Sigma}{\leq} 2\bar{M}(\bar{u})\bar{R}(\bar{u}). \quad (81)$$

This is a key result in our treatment, for it can be interpreted as a constraint on the charge enclosed in Σ . Notice that this constraint exists even if there are curvature singularities at $R = 0$ provided that $m(u, 0) \geq 0$. Ponce de León [46] (see also [5]) obtained a similar result for the much more restricted case of *static, singularity-free* space-times with a charged *perfect fluid* matched to a *pure* Reissner-Nordström exterior. A general result for the general class of static singularity-free metrics can also be deduced from the limits found in [35], and a more recent work is presented in [34]. The fact of considering only static space-times is, possibly, the most important restriction in the above-mentioned papers, since then the results can only be applied to “limiting configurations” for charged static spheres. Only our general approach allows for an interpretation of (81) in terms of the dynamics of the problem and allows for a general analysis of the possible existence of *naked singularities* (see e.g. [45][29] for definitions and relevance) and their creation.

With regard to this, observe that (81) can be also viewed in other interesting ways when expressed as

$$\bar{R}_\Sigma(\bar{u}) \geq \frac{\bar{Q}^2}{2\bar{M}(\bar{u})}.$$

This inequality implies that any space-time \mathcal{V} providing a physically realistic model for the interior of a collapsing star either has its limit surface at values of $\bar{R}_\Sigma(\bar{u}) \geq \bar{Q}^2/(2\bar{M}(\bar{u}))$, or otherwise a *necessarily timelike* singularity⁹ (therefore locally naked) must develop at $R = 0$. The question on whether this radius can effectively be reached lies beyond the purposes of this article, but you can consult [50] and references therein for an interesting discussion at this respect.

There are also constraints on the charge Q of \mathcal{V} due to the fulfillment of DEC. One of them is given by (77). Apart from this, inequalities (27) and (28) define two new possible maximum values for Q : the first one is just the appropriate specialization of (30)

$$Q^2 \leq 2R^2(m_{,R} - R\chi\beta_{,R}), \quad (82)$$

while the second one reads

$$Q^2 \leq \frac{1}{2}R^4(\lambda + P_2). \quad (83)$$

The righthand sides of these two inequalities vanish at $R = 0$ if there is no curvature singularity there, as is clear from (2). Thus, if Σ arrived at the vicinity of $R = 0$ with non-zero charge, the previous conditions would be violated from a certain value

⁹The singularity must be timelike because $m(u, 0) < 0$.

of R on (decreasingly). This result is similar to that in proposition 7.2: Σ can only get arbitrarily close to $R = 0$ if either a curvature singularity already exists, or develops, there.

8 Conclusions

The aim of this work is to provide a theoretical framework for the construction of global models describing stars and voids in General Relativity. Our only assumption is, apart from the spherical symmetry of the spacetime, that the exterior of the star (respectively the gravitational field within the void) is represented by a Vaidya-Reissner-Nordström solution properly matched to an interior (resp. exterior) spacetime with the only restriction that the dominant energy conditions are fulfilled everywhere. Thus, we are treating the general case in which the stars/voids may be charged and radiating.

A summary of our main results is the following. To start with, in proposition 2.2 we saw that the DEC imply the non-negativity of the mass function everywhere if $m(u, 0) \geq 0$. This result has important consequences in the rest of the paper, as for instance the restriction which imposes on any *spherically symmetric* electro-magnetic fields, see section 3. More important consequences are the restrictions on the presence and distribution of electric charge, first in a neighbourhood of $R = 0$ (section 3) and then on the whole spacetime (formulae (82-83) and proposition 7.2).

As a specially important consequence, we have shown in full generality that the *total* charge of any physically acceptable spherically symmetric object has an upper bound related to its size and mass given by (81). Here, by physically acceptable we mean that the object satisfies $m(u, 0) \geq 0$ which in particular includes all bodies which are regular everywhere. This is a result genuine to General Relativity for, as is well known, Classical Electrodynamics does not impose any limitation on the charge that an object can possess. Furthermore, our result is completely general and does not depend on the assumptions of staticity used in its previous partial and particular versions found in [46, 5, 35] and references therein, or in the more recent [34]. In this last work an inequality relating the mass, the radius and the charge of general *static* matter distributions has been found by using the gravitational field equations (in a similar way as Buchdahl [7] obtained the classical maximum mass-radius ratio for uncharged stable stars). Their result shows that a charged ($Q < M$) stable star

must obey the inequality (81). However, as is clear, our treatment provides the limit (81) in dynamical situations, including evolving voids or the important case of collapsing, or rebounding, stars.

The appearance of a maximum value for the charge can be alternatively read as the existence of a minimum value for $R_\Sigma(u)$. This result can be interpreted, for the case of a charged star –no matter how small its charge is– that obeys the DEC as stating that, if the star ever reaches values of R lower than the minimum radius, then a necessarily timelike singularity, ergo locally naked, has to develop at $R = 0$.

All our results have been derived by properly matching the interior and exterior spacetimes, and here we have taken into account the *pure electromagnetic* matching conditions as well as the gravitational ones, which were fully derived in section 5 both in the case with no infinite jumps for the total electromagnetic 4-current — which agrees with experimental results— or in the case with them —this is the usual approach in the literature but, in our opinion, it is unrealistic from a physical point of view, except as an approximation to the real world—. We have also proved that, in generic situations, there is a 2-parameter family of matching hypersurfaces which were later shown to be non-spacelike in Proposition 7.1. The general result that the vast majority of spherically symmetric spacetimes can be matched to a V-RN solution (or one of its particular cases) has a clear physical interpretation given by Theorems 6.1 and 6.2. These results generalize the ones obtained in [16, 39, 2, 42] for several particular cases arising if there is no charge or radiation. However, let us remark that even if the *total* charge of the modeled object is zero and, consequently, the metric for the space-time $\bar{\mathcal{V}}$ is the Vaidya metric (or its specializations), still a radial distribution of charge is allowed in \mathcal{V} , even a time-dependent one, as long as the total charge is zero. This fact allows for a possible description of the charge redistribution phenomena which could take place in the interior of a star. This is the case of, for instance, the *hybrid stars*, in which, despite global charge neutrality, the local non-neutrality is energetically favoured (see [20]).

In our next article (*Spherically symmetric models for charged radiating stars and voids II: Practical approach*) we analyze in depth some of the theoretical formulations proposed in this work and examine some models for stars and voids under the scope of our approach.

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References

- [1] Bekenstein, J.D., *Phys. Rev. D*, **4**, 2185 (1971).
- [2] Bel, L. & Hamoui, A., *Ann. Inst. Henri Poincaré* **A7**, 229 (1967).
- [3] Bekenstein, J.D. & Oron, E., *Phys. Rev.*, **18**, 1809 (1978)
- [4] Bondi, H., *Proc. R. Soc. London*. **A281**, 39 (1964).
- [5] Bonnor, W.B., Cooperstock, F.I., *Phys. Lett. A*, **139** 9, 442 (1989).
- [6] Bonnor, W.B., Oliveira, A.K.G. & Santos, N.O., *Phys. Rep.*, **181**, 269 (1989).
- [7] Buchdahl, H.A., *Phys. Rev.*, **16**, 1027 (1959).
- [8] Cahill, M.E. & McVittie, G.C., *J. Math.Phys.*, **11**, 1382 (1970).
- [9] Carmeli, M. & Kaye, M., *Annals of Physics*, **103**, 97 (1977).
- [10] Carminati, J. & McLenaghan, R.G., *J. Math.Phys.* **32**, 3135 (1991).
- [11] Carot, J. & Tupper, B.O.J., *Phys. Rev. D*, **59**, 124017 (1999).
- [12] Clarke, C.J.S. & Dray, T., *Class. Quantum Grav.*, **4**, 265 (1987).
- [13] Darmon, G., *Mémoires des Sciences Mathématiques*, Gautier-Villars, Paris, Fasc. **25** (1927).
- [14] Einstein, A. & Strauss, E.G., *Rev. Mod. Phys.*, **17**, 120 (1945).
- [15] Fayos, F., Jaen, X., LLanta, E. & Senovilla, J.M.M., *Class. and Quantum Grav.*, **8**, 2057 (1991).

- [16] Fayos, F., Jaen, X., LLanta, E. & Senovilla, J.M.M., Phys. Rev. D, **45**, 2732 (1992).
- [17] Fayos, F., Martín-Prats, M.M. & Senovilla, J.M.M., *Class. Quantum Grav.*, **12**, 2565 (1995).
- [18] Fayos, F., Senovilla, J.M.M. & Torres, R., Phys. Rev. D, **54**, 4862 (1996)
- [19] Florides, P.S., Nuovo Cimento, **42**, 343 (1977).
- [20] Glendenning, N.K., *Compact Stars*. Springer-Verlag. New York (2000).
- [21] Hacyan, S., *Astrophys. J.*, **229**, 42 (1979).
- [22] Hawking, S.W. & Ellis, G.F.R., *The large scale structure of space-time*, Cambridge University Press, Cambridge (1973).
- [23] S.A. Hayward, Phy. Rev. D **53**, 1938 (1996).
- [24] Herrera, L. & Núñez, L., Fundamental Cosmic Physics **14**,235 (1990).
- [25] Israel, W., Nuovo Cimento **44**, 1 (1966).
- [26] Israel, W., Phys. Letters., **24A**, 184 (1967).
- [27] Israel, W., Phys. Rev., **143**, 1016 (1966).
- [28] Jackson, J.D., *Classical Electrodynamics*. John Wiley & Sons, New York, 3rd. Ed. (1999).
- [29] Joshi, P.S., *Global aspects in gravitation and cosmology*, Clarendon Press, Oxford (1993).
- [30] Krori, K.D. & Banerjee, A.J. *J. Phys. A*, **17**, 2125 (1974).
- [31] Lake, K., *Astrophys. J.*, **240**, 744 y **242**, 1238 (1980).
- [32] Lake, K. & Hellaby, C., *Phys. Rev. D*, V.24, 12, p.3019 (1981).
- [33] Lindquist, R.W. & Schwartz, R.A., *Phys. Rev.*, **137B**, 1364 (1965).
- [34] Mak, M.K., Dobson, P.N., Harko, T., *Europhys. Lett.*, **55**, 310 (2001).

- [35] Mars, M., Martín-Prats, M.M., & Senovilla, J.M.M., *Phys. Lett. A*, **218**, 147 (1996).
- [36] Mars, M. & Senovilla, J.M.M., *Class. Quantum Grav.*, **10**, 1865 (1993).
- [37] Martín-Prats, M.M., “*Axially symmetric models for stellar and cosmological structures: black holes, voids, etc.*”. Ph. D. Thesis, University of Barcelona (1995).
- [38] Medina, V., Nuñez, L., Rago, H. & Patiño, A., *Canadian J. Phys.*, **66**, 981 (1988).
- [39] Misner, C.W. & Sharp, D., *Phys. Rev.* **B136**, 571 (1964).
- [40] Misner, C.W., Thorne, K.S. & Wheeler, J.A., *Gravitation*, W.H. Freeman and Company, New York (1970).
- [41] Novikov, E.D., *JEPT Lett.*, **3**, 142 (1966);
 De la Cruz & Israel, W., *Nouvo Cimento* **51.A**, 744 (1967);
 Markov, M.A. & Frolov, V.P., *Theor. Math. Phys.*, **3**, 301 (1970);
 Hamoui, A., *Ann. Inst. Henri Poincaré*, **10**, 195 (1969);
 Vickers, P.A., *Ann. Inst. Henri Poincaré*, **18**, 137 (1973);
 Misra, M.R. & Srivastava, D.C., *Phys. Rev. D* **9**, 844 (1974);
 Raychaudhuri, A.K., *Ann. Inst. Henri Poincaré*, **23**, 229 (1975);
 de Felice, F. & Maeda, K. *Prog. Theor. Phys.*, **68**, 1967 (1982).
- [42] Oliveira, A.K.G. & Santos, N.O., *Ap. J.*, **312**, 640 (1987).
- [43] Oppenheimer, J.R. & Snyder, H., *Phys. Rev.* **56**, 455 (1939).
- [44] Ori, A., *Class. Quantum Grav.*, **7**, 986 (1990).
- [45] Penrose, R., *Riv. Del Nuovo Cimento*, **1** (Numero Spec.), 252 (1969)
 Clarke, C.J.S., *Class. Quantum Grav.*, **10**, 1375 (1993).
 Harada, T., Iguchi, H., Nakao, K.-I., *Prog. Theor. Phys.*, **107**, 449 (2002).
- [46] Ponce de Leon, *Gen. Rel. Grav.* **25**, 1123 (1993).
- [47] Reed B.C. & Henriksen, R.N., *Astrophys. J.*, **236**, 338 (1980).

- [48] Synge, J.L., *Perspectives in geometry and Relativity*, Ed. B. Hoffmann, (Indiana U.P., Bloomington, 1966).
- [49] Synge, J.L., *Relativity: The general theory*. North-holland Publishing Company. (Amsterdam, 1966).
- [50] Tipler, F.J. *Phys. Rev. D* **16**, 3359 (1977);
Gursel, Y., Sandberg, V.D., Novikov, I.D., and Starobinsky, A.A. *Phys. Rev. D* **19**, 413 (1979);
Gursel, Y., Novikov, I.D., Sandberg, V.D., and Starobinsky, A.A. *Phys. Rev. D* **20**, 1260 (1979);
Chandrasekhar, S., and Hartle, J.B., *Proc. R. Soc. London A* **384**, 301 (1982);
Yurtsever, U., *Phys. Rev. D* **36**, 1662 (1987);
Israel, W., in *Black Holes and Relativistic Stars*. The University of Chicago Press. Ed. R.M. Wald. 137 (1998).
- [51] Torres, R., *Construcción de modelos de estrellas y burbujas de vacío o radiativas, en el marco de la Relatividad General: Un planteamiento global*. Ph. D. Thesis, Universitat Politècnica de Catalunya, Barcelona, (2001). (PS formatted version available on demand at rtorresh@wanadoo.es)
- [52] Vaidya, P.V., *Proc. Indian Acad. Sci.* **A33**, 264 (1951).
- [53] Wald, R.M., *General Relativity*, The University of Chicago Press, Chicago (1984).
- [54] Waugh, B. & Lake, K., *Phys.Rev.D*, **34**, 2978 (1986).
- [55] Weinberg, S., *Gravitation and Cosmology: Principles and applications of the general theory of relativity*, John Wiley & Sons, New York (1972).